



Sistemler ve Sinyaller



Ders 6

Konvolusyon

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DS Sinyallerin cevabı- Konvolüsyon

- Konvolüsyon şu şekilde hesaplanır:

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau = \int_{-\infty}^{\infty} h(\tau)x(t - \tau)d\tau$$

- Sinyal değişkenini τ olarak belirleyelim
- Fonksiyonu Y eksenini üzerinde ters çevirelim
- T kadar kaydıralım
- İki sinyali çarpalım
- İçi çarpımın altında kalan alanı hesaplayalım(integration)

Örnek 2.6

$x(t)$, birim dürtü tepkisi $h(t)$ olan bir LTI sisteminin girdisi olsun, burada

$$x(t) = e^{-at}u(t), \quad a > 0$$

Ve

$$h(t) = u(t).$$

1. $h(\tau)$ ve $x(\tau)$ fonksiyonlarını ele alalım
2. $h(\tau)$ 'yi ters çevirin ve "t" ile kaydırın
3. Not edin, $t < 0$ için, $x(\tau)$ ve $h(t - \tau)$ 'nin çarpımı sıfırdır ve sonuç olarak $y(t)$ sıfırdır. $t > 0$ olduğunda

$$x(\tau)h(t - \tau) = \begin{cases} e^{-a\tau}, & 0 < \tau < t \\ 0, & \text{otherwise} \end{cases}$$

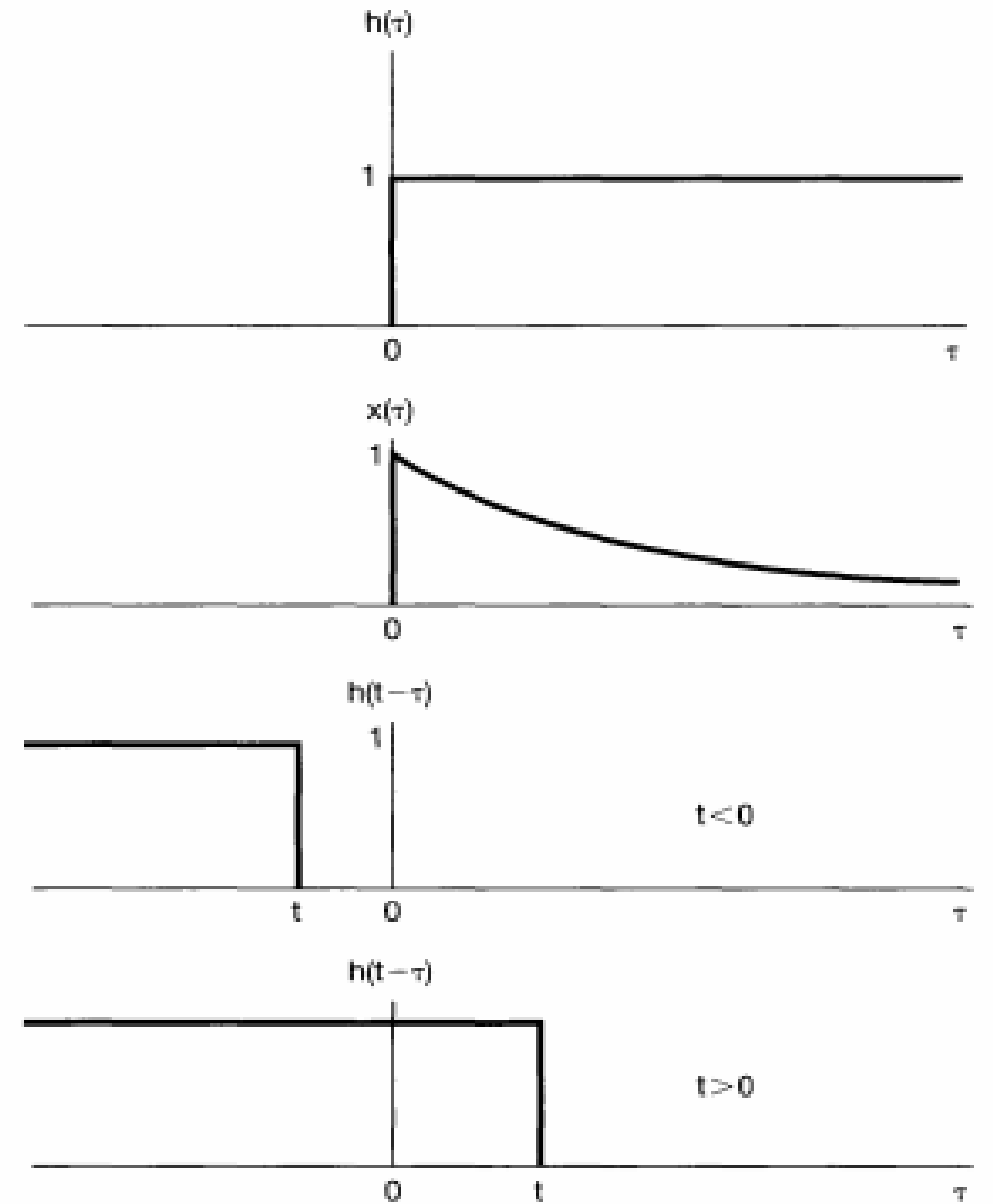


Figure 2.17 Calculation of the convolution integral for Example 2.6.

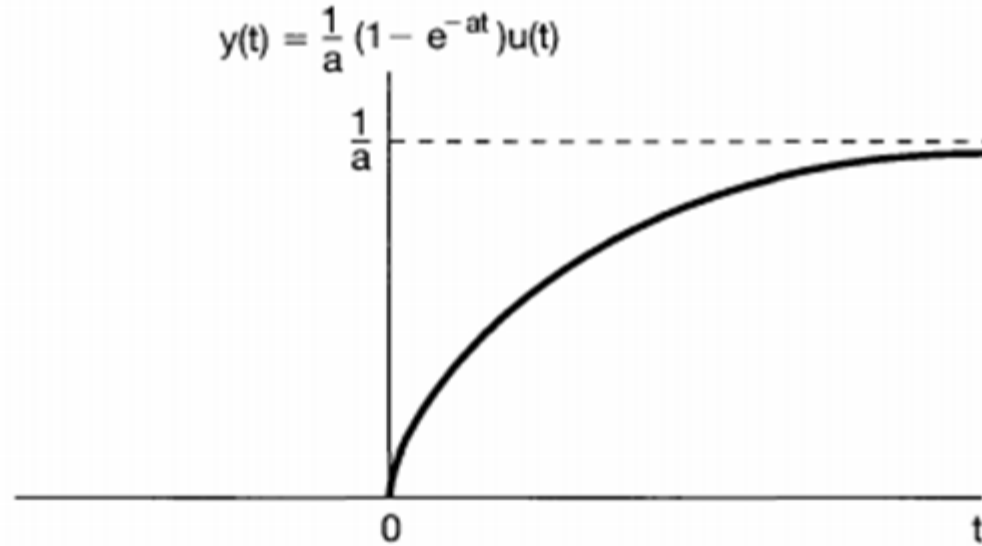
Bu ifadeden $t > 0$ için $y(t)$ 'yi hesaplayabiliriz:

$$y(t) = \int_0^t e^{-a\tau} d\tau = \frac{1}{a}(1 - e^{-at}).$$

Bu nedenle, tüm t için $y(t)$

$$y(t) = \frac{1}{a}(1 - e^{-at})u(t),$$

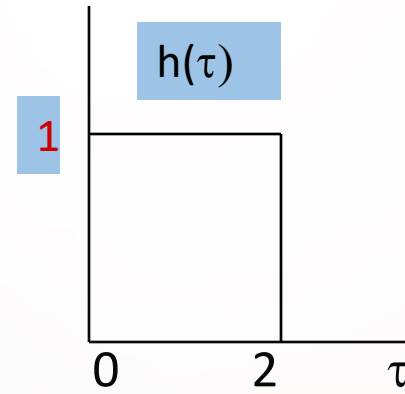
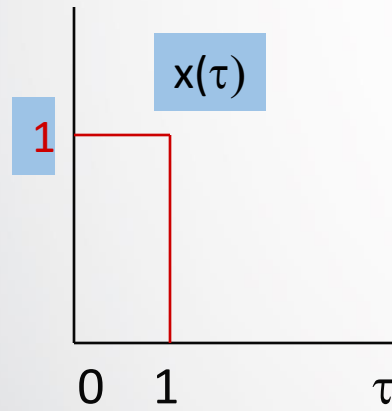
Şekil 2.18'de gösterilmiştir.



Şekil 2.18 Örnek 2.6'daki sistemin, $x(t) = e^{-at}u(t)$ girişine olan darbe tepkisi $h(t) = u(t)$ ile tepkisi.

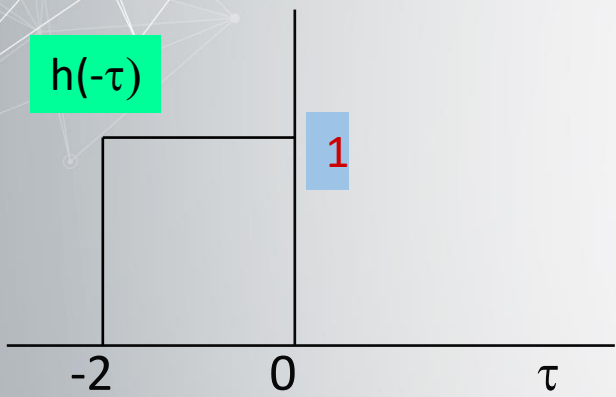
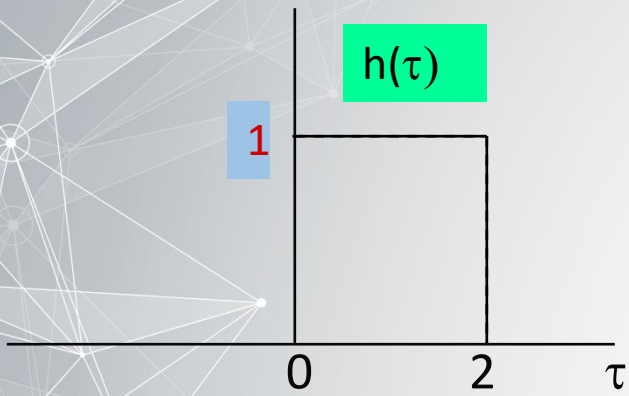
Örnek

- Her biri 1 yüksekliğinde iki darbenin evrişimi

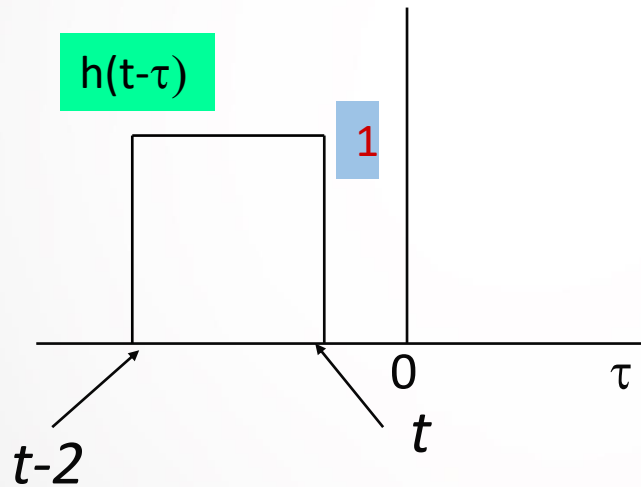


$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau$$

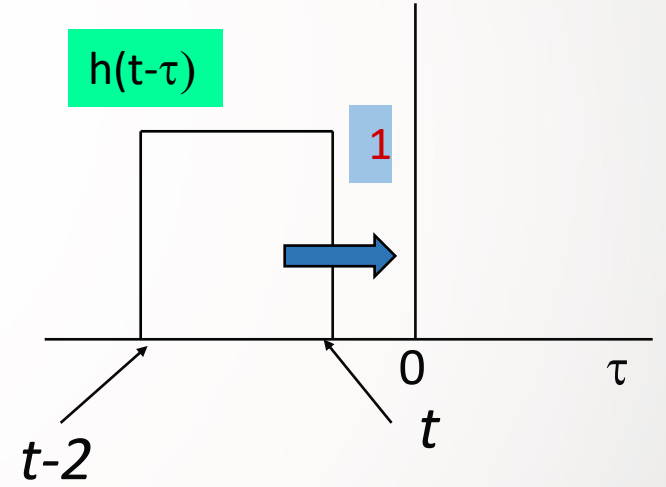
Example



1. Ters fonksiyon

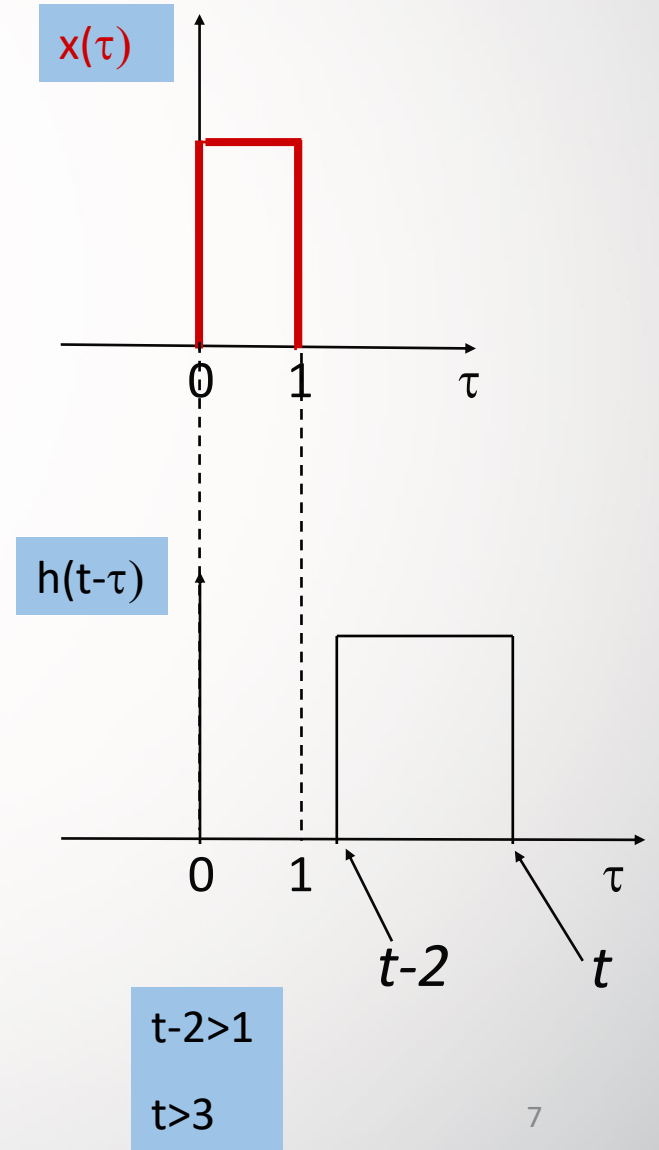
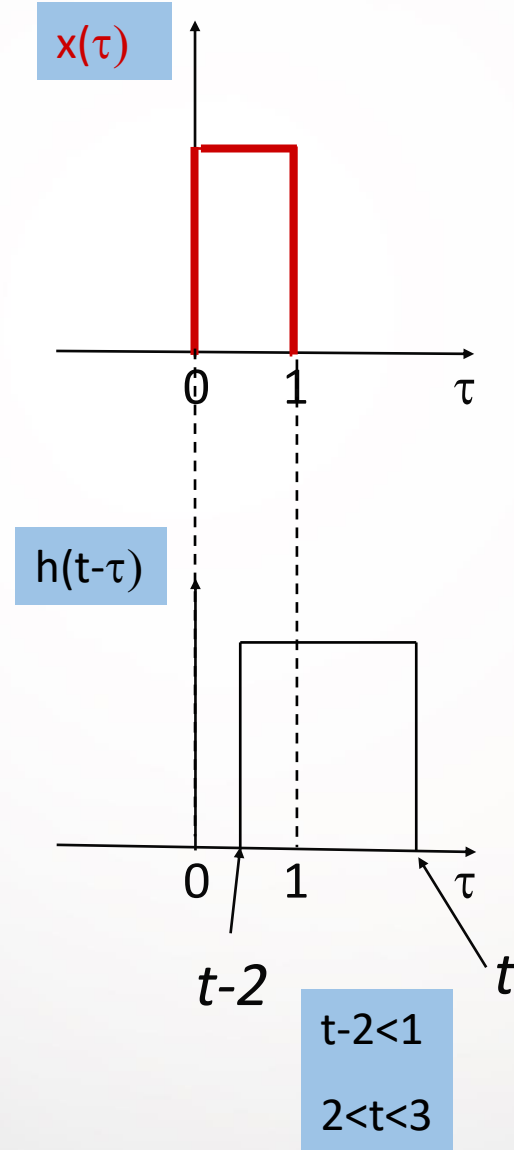
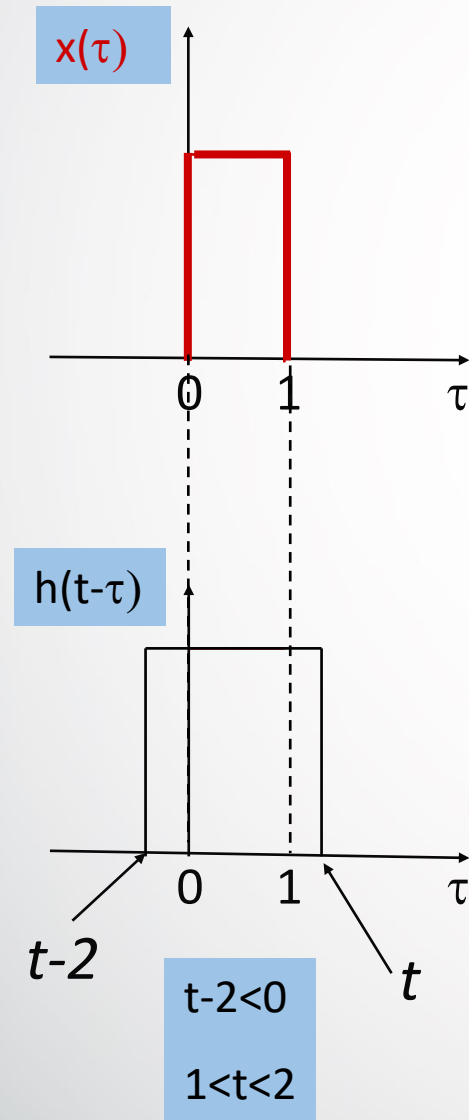
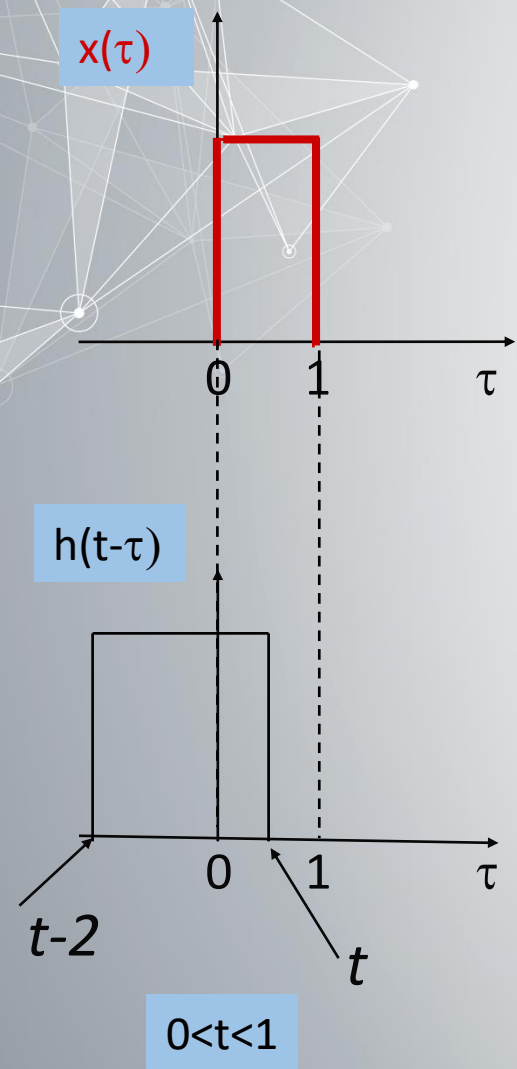


2. Shift fonksiyonu $h(-\tau)$

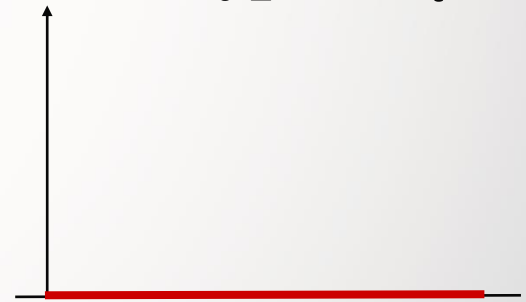
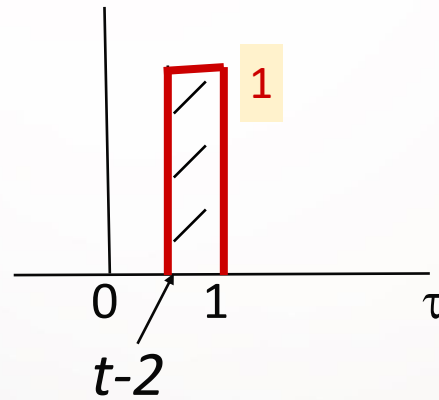
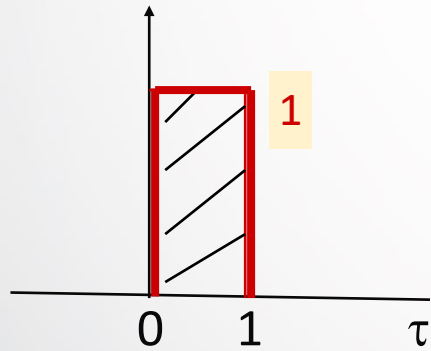
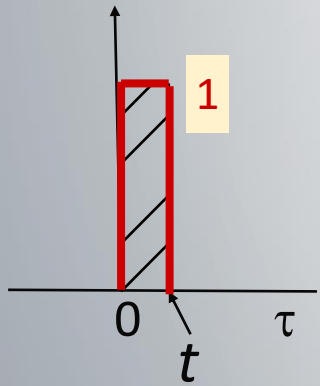
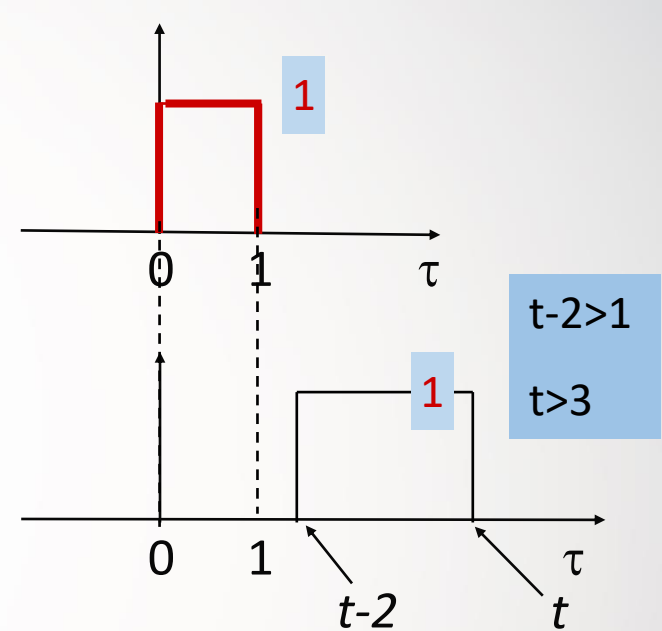
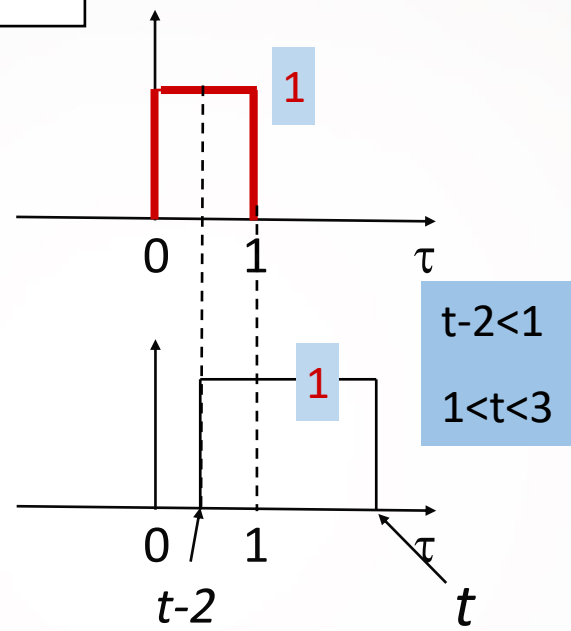
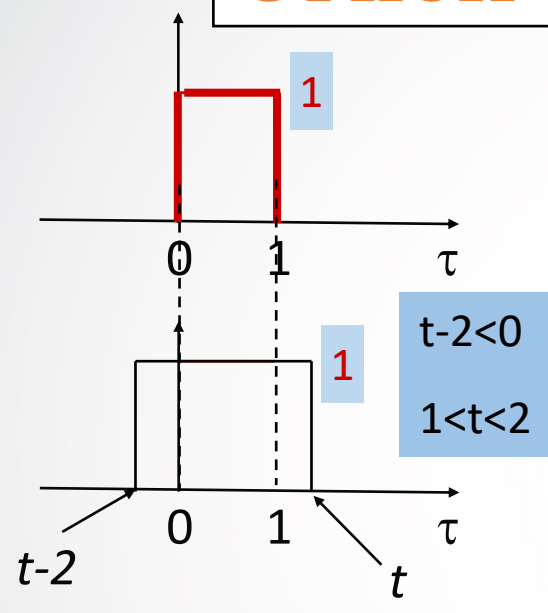
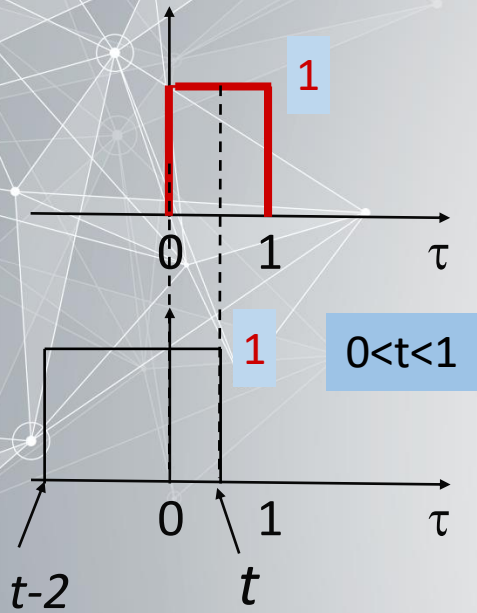


3. 't'yi artırın

Örnek



Örnek



$$y(t) = \int_0^t d\tau = t$$

$$y(t) = \int_0^1 d\tau = 1$$

$$y(t) = \int_{t-2}^1 d\tau = 3 - t$$

$$y(t) = 0$$

Örnek

$$y(t) = \begin{cases} t & 0 < t < 1 \\ 1 & 1 < t < 2 \\ 3 - t & 2 < t < 3 \\ 0 & t > 3 \end{cases}$$

