

SİNYALLER VE SİSTEMLER

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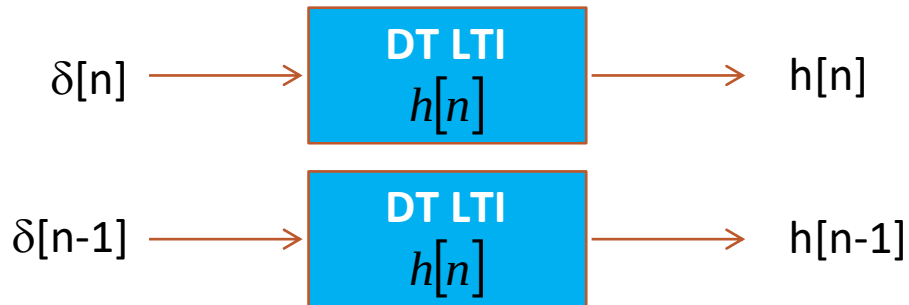


DOĞRUSAL ZAMANLA DEĞİŞMEYEN SİSTEMLER (LTI)

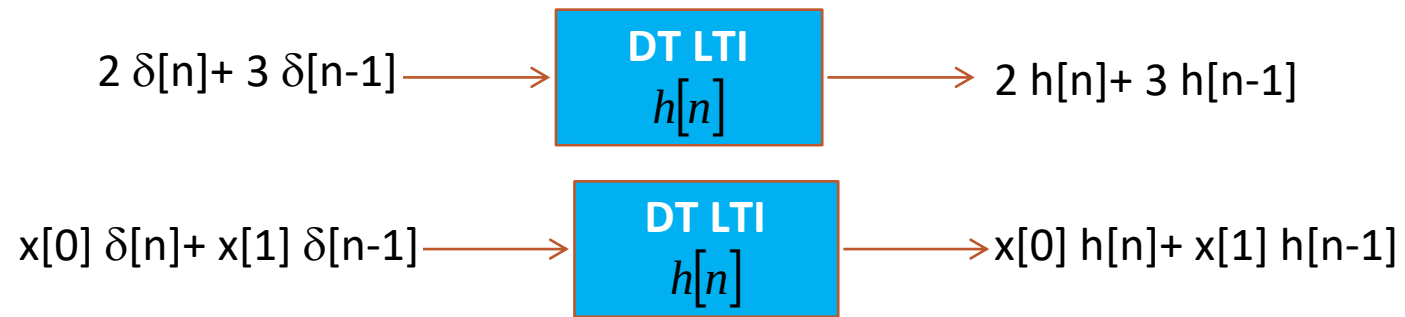
- Birçok fiziksel süreç, doğrusal zamanla değişmez (LTI) sistemler olarak modellenenebilir.
- Dürtü tepkisini tanımlayın, $h[n]$, bir DT LTI sisteminin bir birim darbe fonksiyonuna tepkisi olarak, $\delta[n]$.



- Zaman değişmezliği ilkesini kullanarak:



- Doğrusallık ilkesini kullanarak:



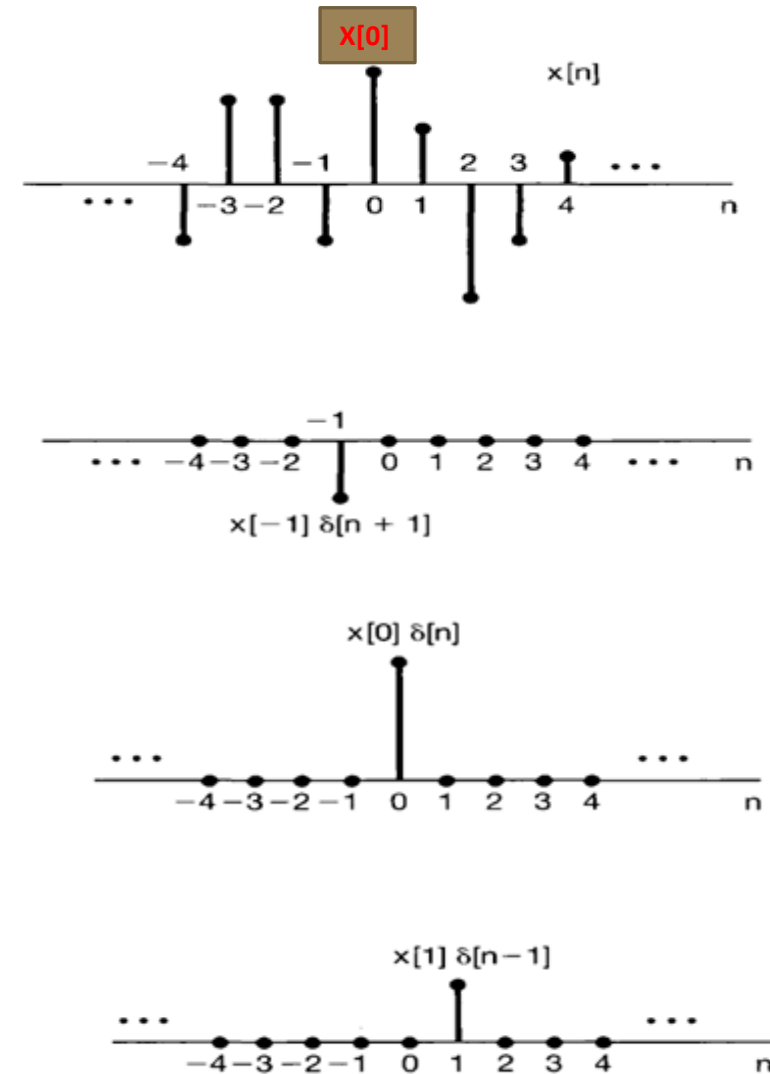
DT Sinyallerinin Gösterimi

Dizilerin toplamı $x[n]$ 'ye eşittir.

$$x[n] = \dots + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + \dots$$

Kompakt bir form

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k].$$



Bir DT Sisteminin Tepkisi- Konvolusyon (Convolution)



Giriş/çıkış ilişkisi

Doğrusallık

$$x[n] = \dots + x[-1] \delta[n+1] + x[0] \delta[n] + x[1] \delta[n-1] + \dots$$

$$y[n] = \dots + x[-1] h[n+1] + x[0] h[n] + x[1] h[n-1] + \dots$$

Kompakt bir form

```
graph LR; x[n] --> block[DT LTI h[n]]; block --> y[n];
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$$x[n] = \sum_{k=-\infty}^{\infty} x[k] \delta[n-k] \rightarrow \text{DT LTI } h[n] \rightarrow y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

Bir DT Sisteminin Tepkisi- Konvolusyon (Convolution)



convolution operator

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k] = x[n] * h[n]$$

convolution sum

Toplama, evrişim toplamı olarak adlandırılır.

Evrişim işlemini belirtmek için "*" sembolü kullanılır.

Herhangi bir DT LTI'nin çıkışı, giriş sinyalinin darbe tepkisi ile bir evrişimdir

Konvolüsyon

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

1. Analitik Yöntem

$$y[n] = \dots + x[-1] h[n+1] + x[0] h[n] + x[1] h[n-1] + \dots$$

$x[0]$ ve $x[1]$ 'in sıfırdan farklı olduğunu varsayalım

$$y[n] = x[0] h[n] + x[1] h[n-1]$$

2. Grafiksel Yöntem

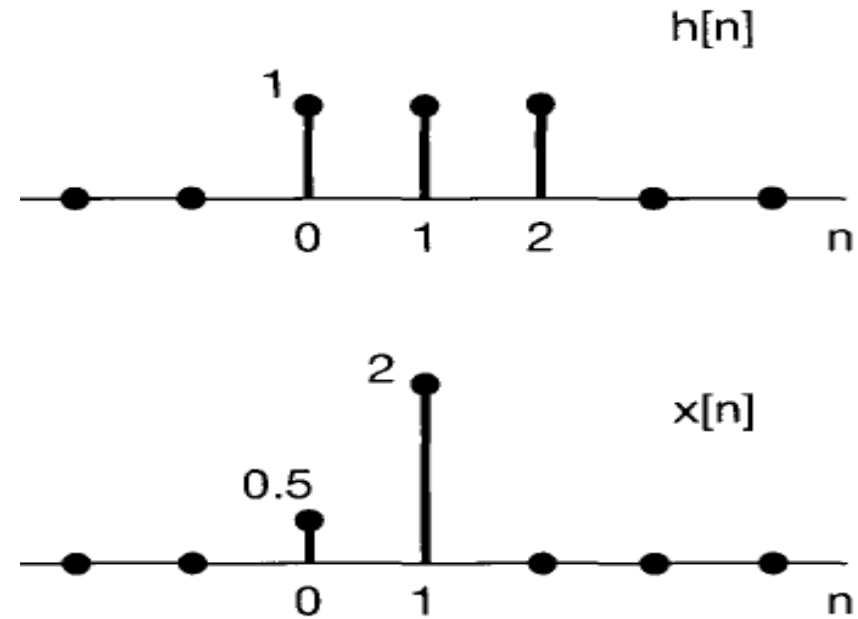
$$\begin{aligned} h[k] &\xrightarrow{\text{Flip}} h[-k] \\ &\xrightarrow{\text{Shift}} h[n-k] \\ &\xrightarrow{\text{Multiply}} x[k] h[n-k] \\ &\xrightarrow{\text{Sum}} \sum_{k=-\infty}^{+\infty} x[k] h[n-k] \end{aligned}$$

Örnek 2.1

$y[n]$ 'yi bulun ve $x[n]$, $h[n]$ ve $y[n]$ 'yi çizin

$$h[n] = \delta[n] + \delta[n - 1] + \delta[n - 2]$$

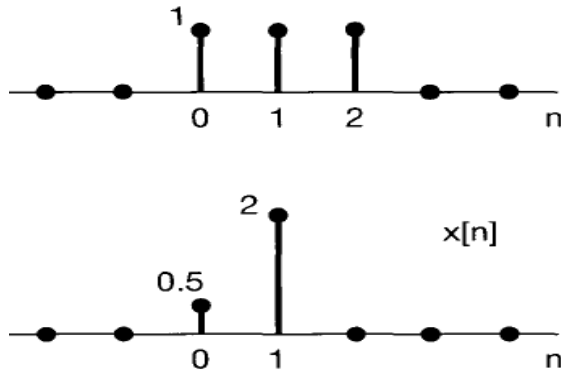
$$x[n] = 0.5 \delta[n] + 2 \delta[n - 1]$$



1. Analitik Yöntem

Örnek 2.1

Darbe tepkisi $h[n]$ ve giriş $x[n]$ olan bir LTI sistemini ele alalım,



Bu durumda, yalnızca $x[0]$ ve $x[1]$ sıfır olmadığından,

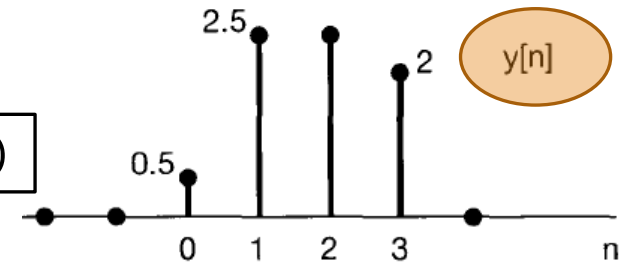
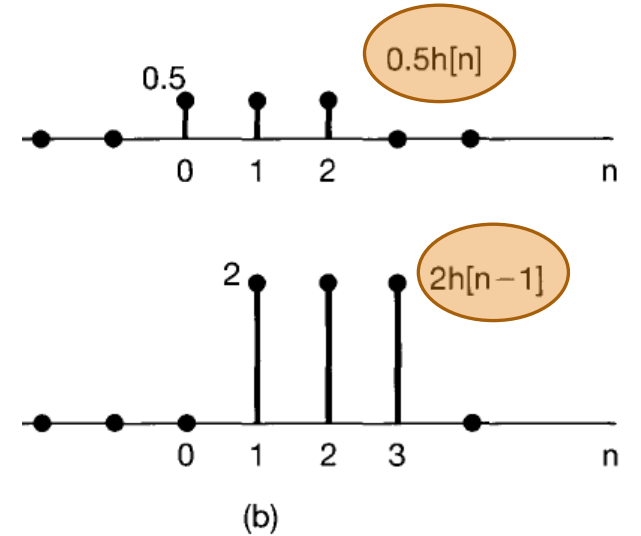
$$y[n] = x[0]h[n - 0] + x[1]h[n - 1]$$

$$= 0.5h[n] + 2h[n - 1].$$

$$y[n] = \mathbf{0.5} * (\delta[n] + \delta[n - 1] + \delta[n - 2]) + \mathbf{2} (\delta[n - 1] + \delta[n - 2] + \delta[n - 3])$$

$$y[n] = 0.5 \delta[n] + 2.5 \delta[n - 1] + 2.5 \delta[n - 2] + 2 \delta[n - 3]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n - k]$$



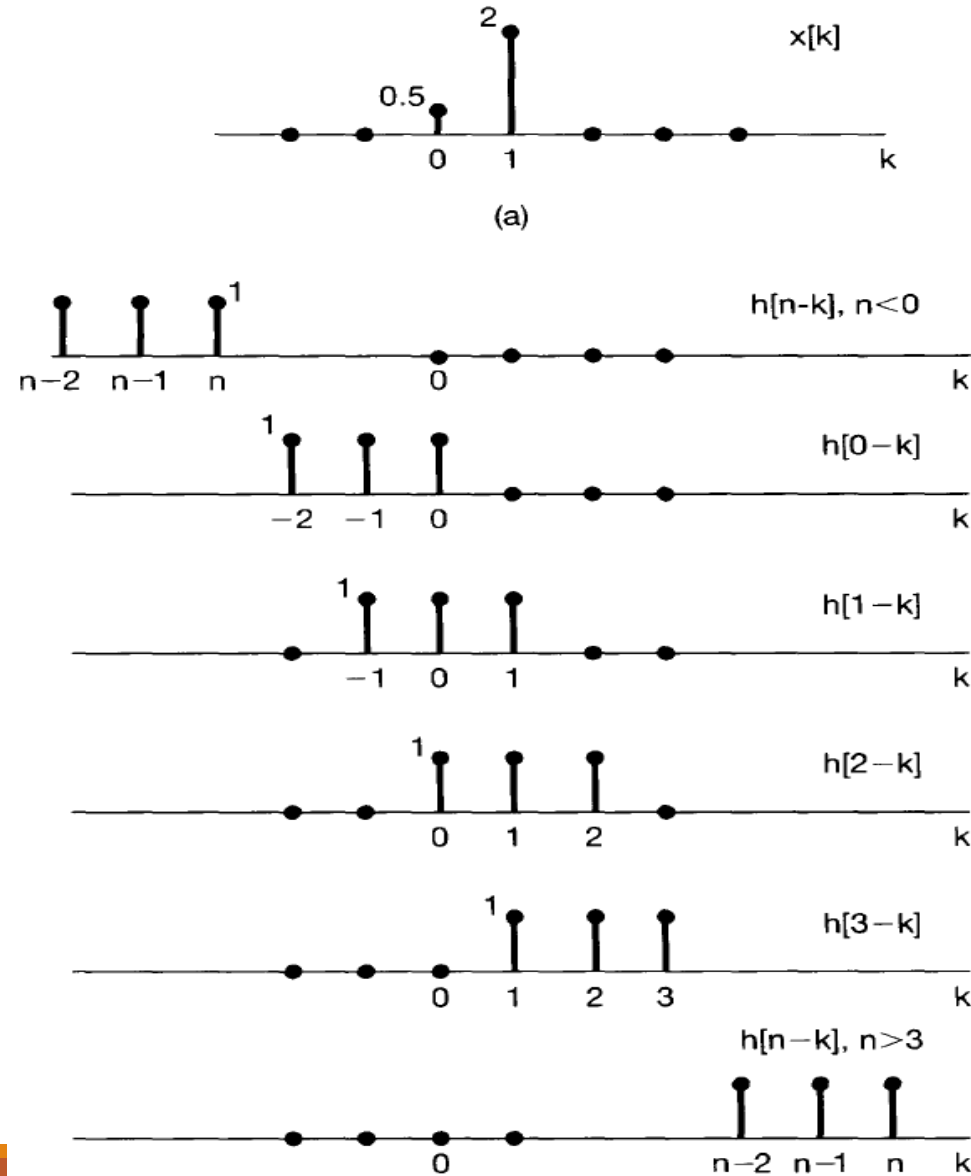
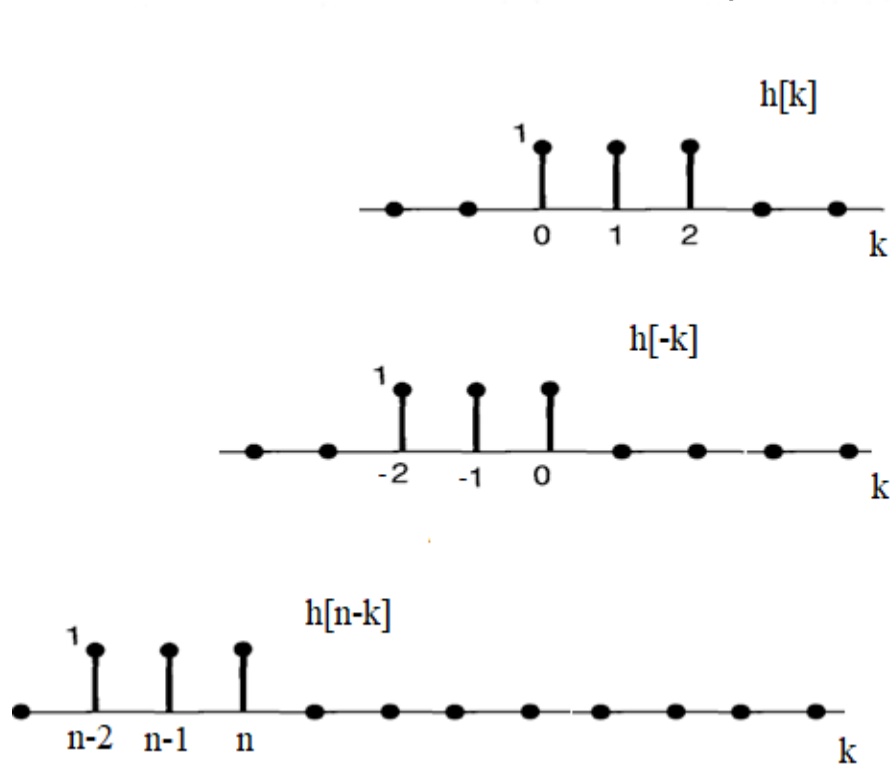
2. Grafiksel Yöntem

Örnek 2.1

Darbe tepkisi $h[n]$ ve giriş $x[n]$ olan bir LTI sistemini ele alalım,

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

$$\begin{aligned} h[k] &\xrightarrow{\text{Flip}} h[-k] \\ &\xrightarrow{\text{Shift}} h[n-k] \\ &\xrightarrow{\text{Multiply}} x[k]h[n-k] \\ &\xrightarrow{\text{Sum}} \sum_{k=-\infty}^{+\infty} x[k]h[n-k] \end{aligned}$$



$$\Rightarrow y[0] = 0.5$$

$$\Rightarrow y[1] = 2.5$$

$$\Rightarrow y[2] = 2.5$$

$$\Rightarrow y[3] = 2$$



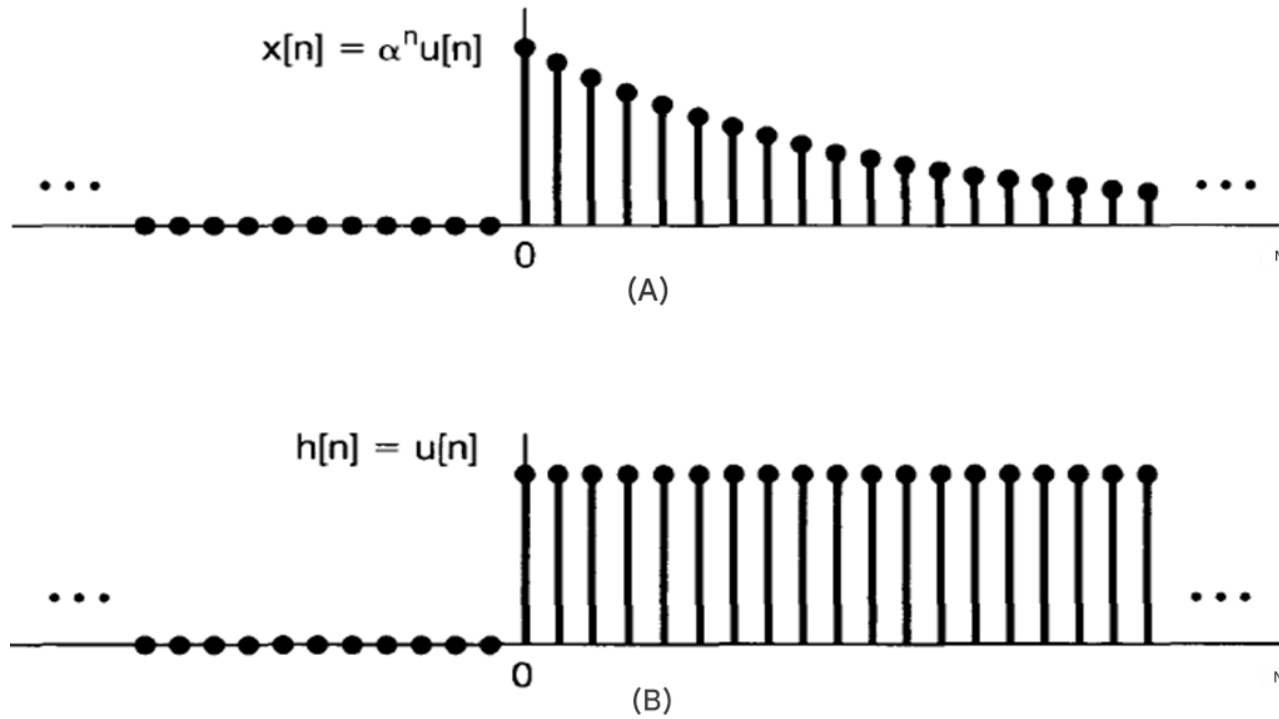
Örnek 2.3

$x[n]$ girişini ve $h[n]$ birim dürtü tepkisini aşağıdaki şekilde verildiğini düşünün:

$$x[n] = \alpha^n u[n],$$

$$h[n] = u[n],$$

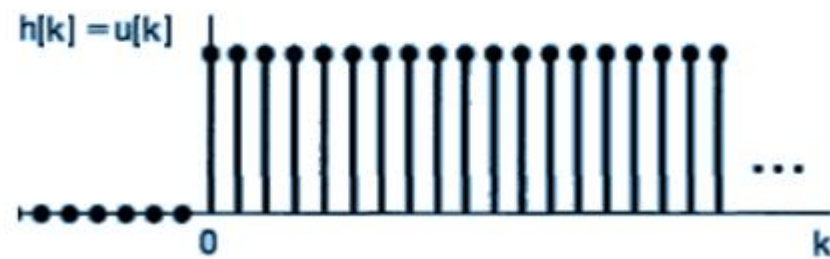
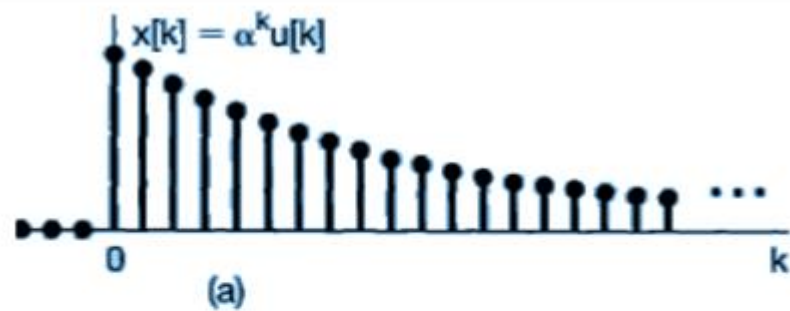
$0 < \alpha < 1$ ile. < Bu sinyaller Şekil'de gösterilmiştir



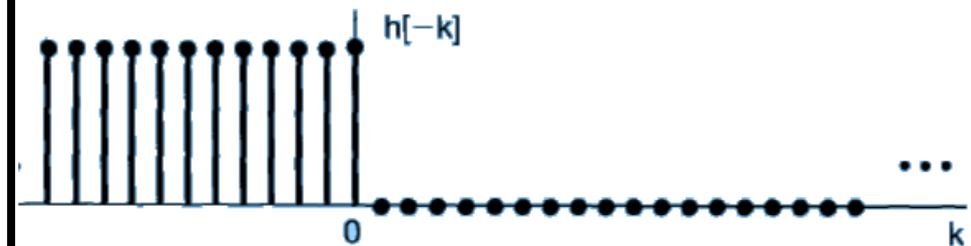
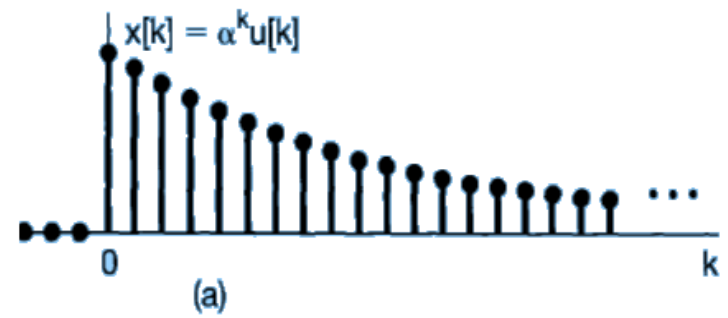
Şekil 2.5 Örnek 2.3'teki $x[n]$ ve $h[n]$ sinyalleri.

Örnek 2.3

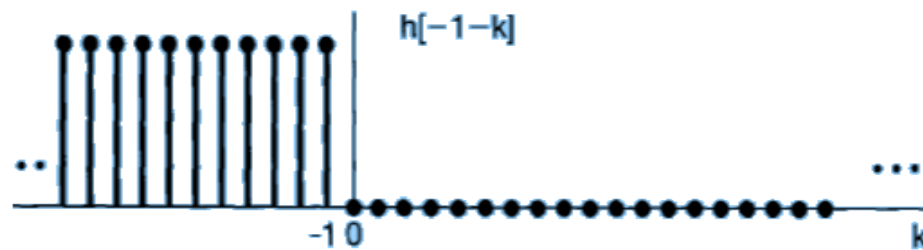
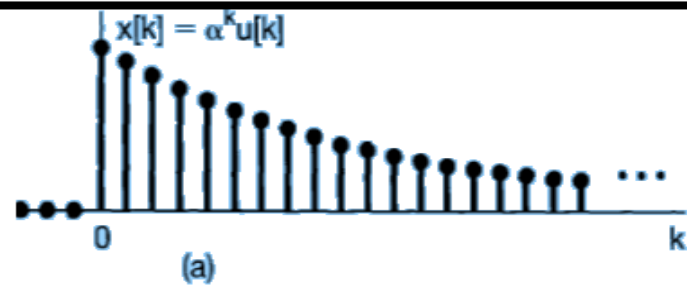
1



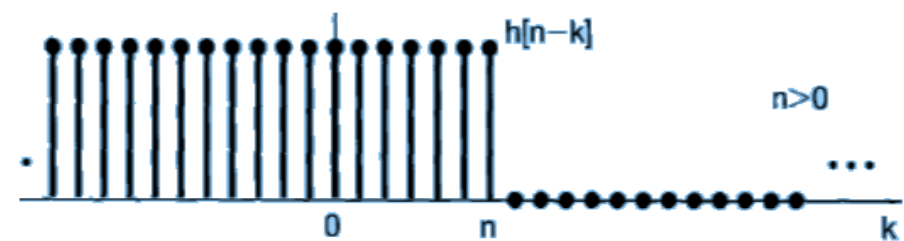
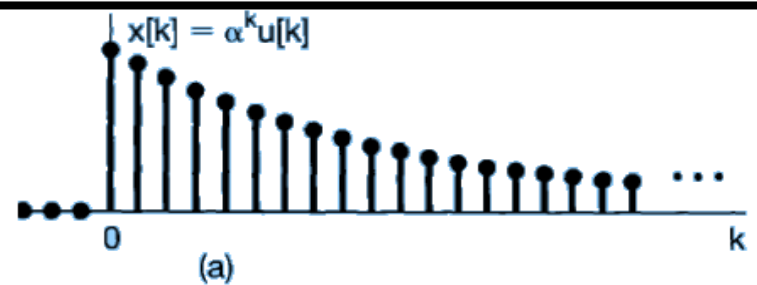
2

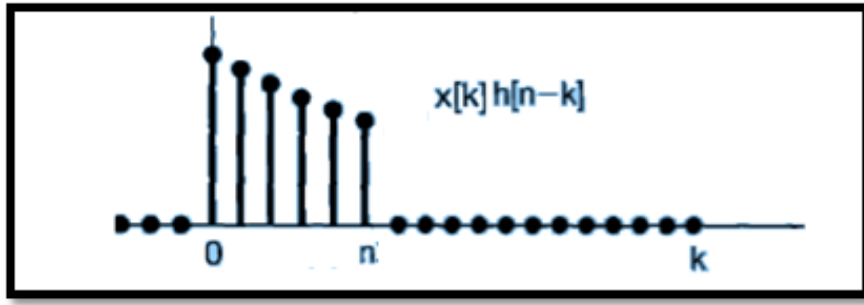


3



4





$$x[k]h[n-k] = \begin{cases} \alpha^k, & 0 \leq k \leq n \\ 0, & \text{değilse} \end{cases}$$

$$\sum_0^n x[k] h[n-k] = 1 + \alpha^1 + \alpha^2 + \alpha^3 + \dots + \alpha^n$$

Böylece : $n \geq 0$,

$$y[n] = \sum_{k=0}^n \alpha^k = \frac{1 - \alpha^{n+1}}{1 - \alpha}$$

$$y[n] = \left(\frac{1 - \alpha^{n+1}}{1 - \alpha} \right)$$

